

**Versão UK**

**Scientific Work Proposal for Mathematics Phd Thesis**

**Scientific Domain**

Mathematics

**Main Area**

Mathematics of Fluid Mechanics and its Applications, Analysis Mathematics, Numerical Simulation and Mathematical Modeling with Experimentations

**Thesis title (Provisory)**

One-dimensional Fluid-Structure Interaction Problems using the Cosserat Theory

**Keywords**

One-dimensional model, Fluid-structure Interaction Problems, Cosserat Theory, Newtonian Fluids, Non-Newtonian Fluids, Partial Differential Equations, Ordinary Differential Equations, Laboratory Experimentation.

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**Scientific Work Proposal:**

In recent decades, the study of the phenomena of interaction between fluid and structure have attracted a large number of researchers from different fields such as mechanical engineering, biology, medicine, biomedical engineering, physics and mathematics. In the field of applications in medicine, biology and medical engineering, the phenomena of fluid-structure have received increasing attention in the scientific community because they have application in a wide variety of problems associated with the flow of blood in the vascular system as, for example, in the clinical problem of an arteriosclerosis or aneurysm vein situation, among others. Today, the study of phenomena of fluid-structure is a topic of great interest, with major social and economic implications. The models to be studied are related to the specific problem associated with the arteriosclerosis or aneurysm vein situation, i.e., involving the flow of blood, coupled to structure and/or permeability of the vein. The most recent mathematical models for this fluid-structure interaction problem found in the literature are sophisticated, with its genesis in a wide variety of factors, such as multiple scales and complex interactions between fluid and porous medium. These models usually involve coupled systems of nonlinear, time-dependent partial (or ordinary) differential equations and algebraic equations. Its analytical and numerical treatment is complex and requires the use of sophisticated arguments. In order to overcome this deviation, we use an alternative theory, the theory of Cosserat associated with fluid dynamics, which allow us to approximate the 3D model by simplified 1D models. Based on this approach theory we reduce the nonlinear three-dimensional equations governing the axisymmetric unsteady motion of a non-Newtonian incompressible fluid (or Newtonian fluid) to a one-dimensional system of ordinary (or partial) differential equations depending on time and on a single spatial variable. From this new system we obtain the unsteady equation for the mean pressure gradient and the wall shear stress both depending on the volume flow rate, Womersley number, viscosity and viscoelastic parameters over a finite section of a straight tube with variable circular cross-section. The aim of this project is precisely to study, in terms of mathematical analysis, numerical optimization and laboratory experiences these alternative models resulting from the application of the Cosserat theory to the clinical problem of arteriosclerosis or aneurysm vein situation.

#### **Some General References about Fluid-Structure Interaction Problems:**

- [1] K. J. Paik, Simulation of fluid-structure interaction for surface ships with linear/nonlinear deformations [Ph.D. thesis], University of Iowa, 2010.
- [2] Z. Mortazavinia, A. Zare, and A. Mehdizadeh, "Effects of renal artery stenosis on realistic model of abdominal aorta and renal arteries incorporating fluid-structure interaction and pulsatile non-Newtonian blood flow," *Applied Mathematics and Mechanics*, vol. 33, no. 2, pp. 165–176, 2012.
- [3] W. Zhang, C. Herrera, S. N. Atluri, and G. S. Kassab, "Effect of surrounding tissue on vessel fluid and solid mechanics," *Journal of Biomechanical Engineering*, vol. 126, no. 6, pp. 760–769, 2004.

- [4] V. Kanyanta, A. Ivankovic, and A. Karac, "Validation of a fluid-structure interaction numerical model for predicting flow transients in arteries," *Journal of Biomechanics*, vol. 42, no. 11, pp. 1705–1712, 2009.
- [5] P. K. Siogkas, A. I. Sakellarios, T. P. Exarchos et al., "Blood flow in arterial segments: rigid vs. deformable walls simulations," *Journal of the Serbian Society for Computational Mechanics*, vol. 5, no. 1, pp. 69–77, 2011.
- [6] M. H. Friedman, R. Krams, and K. B. Chandran, "Flow interactions with cells and tissues: cardiovascular flows and fluid-structure interactions: sixth international bio-fluid mechanics symposium and workshop, March 28–30, 2008, Pasadena, California," *Annals of Biomedical Engineering*, vol. 38, no. 3, pp. 1178–1187, 2010.
- [7] K. W. Lee and X. Y. Xu, "Modelling of flow and wall behaviour in a mildly stenosed tube," *Medical Engineering and Physics*, vol. 24, no. 9, pp. 575–586, 2002.
- [8] S. Badia, A. Quaini, and A. Quarteroni, "Splitting methods based on algebraic factorization for fluid-structure interaction," *SIAM Journal on Scientific Computing*, vol. 30, no. 4, pp. 1778–1805, 2008.
- [9] G. Guidoboni, R. Glowinski, N. Cavallini, S. Čanić, and S. Lapin, "A kinematically coupled time-splitting scheme for fluid-structure interaction in blood flow," *Applied Mathematics Letters*, vol. 22, no. 5, pp. 684–688, 2009.
- [10] F. Nobile and C. Vergara, "An effective fluid-structure interaction formulation for vascular dynamics by generalized Robin conditions," *SIAM Journal on Scientific Computing*, vol. 30, no. 2, pp. 731–763, 2008.

**Some Preparatory References Works from the Advisor, Co-Advisor and from Professor Ashwin Vaidya:**

- [1] Carapau, F., and Sequeira, A., 1D Models for Blood Flow in Small Vessels Using the Cosserat Theory, *WSEAS Transactions on Mathematics*, Issue 1, Volume 5, pp. 54-62, 2006.
- [2] Carapau, F., and Sequeira, A., Axisymmetric motion of a second order viscous fluid in a circular straight tube under pressure gradients varying exponentially with time, *WIT Transactions on Engineering Sciences*, Vol.52, pp. 409-419, 2006.
- [3] Carapau, F., and Sequeira, A., Unsteady flow of a generalized Oldroyd-B fluid using a director theory approach, *WSEAS Transactions on Fluid Mechanics*, Issue 2, Volume 1, pp. 167-174, 2006.

- [4] Janela, J., Sequeira, A., and Carapau, F., Numerical Simulation of the Motion of Rigid Particles in Generalized Newtonian Fluids using a Hyper-Viscosity Method, WSEAS Transactions on Mathematics, Issue 4, Volume 5, pp. 366-373, 2006.
- [5] Carapau, F., Sequeira, A., and Janela, J., 1D simulations of second-grade fluids with shear-dependent viscosity, WSEAS Transactions on Mathematics, Issue 1, Volume 6, pp. 151-158, 2007.
- [6] Carapau, F., and Sequeira, A., 1D dynamics of a second-grade viscous fluid in a constricted tube, Banach Center Publications, Inst. Math, Polish Acad. Sc., Volume 81, pp. 95-103, 2008.
- [7] Carapau, F., Axisymmetric motion of a generalized Rivlin-Ericksen fluids with shear-dependent normal stress coefficients, Inter. Journal of Mathematical Models and Methods in Applied Sciences, Issue 2, Volume 2, pp. 168-175, 2008.
- [8] Carapau, F., Analysis of perturbed flows of a second-order fluid using a 1D hierarchical model, Inter. Journal of Mathematics and Computers in Simulation, Issue 3, Volume 2, pp. 256-263, 2008.
- [9] Carapau, F., Axisymmetric Swirling Motion of Viscoelastic Fluid Flow Inside a Slender Surface of Revolution, IAENG Engineering Letters, Issue 4, Volume 17, pp. 238-245, 2009.
- [10] Carapau, F., 1D Viscoelastic Flow in a Circular Straight Tube with Variable Radius, Int. J. Appl. Math. Stat., No. D10, Volume 19, pp. 20-39, 2010.
- [11] Carapau, F., One-dimensional viscoelastic fluid model where viscosity and normal stress coefficients depend on the shear rate, Nonlinear Analysis: Real World Applications, Volume 11, pp. 4342-4354, 2010.
- [12] Carapau, F., and Janela, J., A one-dimensional model for unsteady axisymmetric swirling motion of a viscous fluid in a variable radius straight circular tube. International Journal of Engineering Science, Volume 72, pp. 107-116, 2013.
- [14] Carapau, F., and Correia, P., Numerical simulations of a third-grade fluid flow on a tube through a contraction, European Journal of Mechanics B/Fluids, V. 65, pp. 45-53, 2017.
- [15] Fernando Carapau, Paulo Correia, Luís M. Grilo and Ricardo Conceição, Axisymmetric Motion of a Proposed Generalized Non-Newtonian Fluid Model with Shear-dependent Viscoelastic Effects, IAENG International Journal of Applied Mathematics, vol. 47, no. 4, pp. 361-370, 2017.

[16] Carapau, F., Janela, J., Correia, P., Vila, S., Numerical Solvability of a Cosserat Model for the Swirling Motion of a Third-grade Fluid in a Constant Radius Straight Circular Tube, *Int. J. Appl. Math. Stat.*, vol. 57, no. 2, pp. 1-15, 2018.

[17] Carapau, F., Conceição, R., Three-Dimensional Velocity Field for Blood Flow Using the Power-Law Viscosity Function, *WSEAS Transactions on Heat and Mass Transfer*, V.13, pp. 35-48, 2018.

[18] P. Areias, T. Rabczuk, F. Carapau and J. Carrilho Lopes, A continuous-stress tetrahedron for finite strain problems, *Finite Elements in Analysis and Design*, vol. 165, pp. 52-64, 2019.

[19] P. Areias, C. Tiago, J. Carrilho Lopes, F. Carapau, P. Correia, A finite strain Raviart-Thomas tetrahedron, *European Journal of Mechanics / A Solids*, V. 80 (2020) 103911 (doi.org/10.1016/j.euromechsol.2019.103911)

[20] F. Carapau, P. Correia, T. Rabczuk and P. Areias, One-dimensional model for the unsteady flow of a generalized third-grade viscoelastic fluid, *Journal Neural Computing & Applications*, (DOI: 10.1007/s00521-020-04733-w)

[21] Corrigendum to “On the modified least action principle with damping” [*Eur. J. Mech. B Fluids* 89 (2021) 301-311](S099775462100090X)(10.1016/j.euromechflu.2021.06.005), Fitzgerald, K., Massoudi, M. & Vaidya, A., 1 Jan 2022, In: *European Journal of Mechanics, B/Fluids*. 91

[22] Network Thermodynamics-Based Scalable Compartmental Model for Multi-Strain Epidemics, Pateras, J., Vaidya, A. & Ghosh, P., Oct 2022, In: *Mathematics*. 10, 19, 3513

[23] On the Thermodynamics of Self-Organization in Dissipative Systems: Reflections on the Unification of Physics and Biology, Chung, B. J., De Bari, B., Dixon, J., Kondepudi, D., Pateras, J. & Vaidya, A., Apr 2022, In: *Fluids*. 7, 4, 141

[24] Ramos, C., Carapau, F., Correia, P. (2022). Cellular Automata Describing Non-equilibrium Fluids with Non-mixing Substances. In: Carapau, F., Vaidya, A. (eds) *Recent Advances in Mechanics and Fluid-Structure Interaction with Applications. Advances in Mathematical Fluid Mechanics*. Birkhäuser, Cham. [https://doi.org/10.1007/978-3-031-14324-3\\_10](https://doi.org/10.1007/978-3-031-14324-3_10)

[25] Carapau, F., Correia, P., Areias, P. (2022). Three-Dimensional Velocity Field Using the Cross-Model Viscosity Function. In: Carapau, F., Vaidya, A. (eds) *Recent Advances in Mechanics and Fluid-Structure Interaction with Applications. Advances in Mathematical Fluid Mechanics*. Birkhäuser, Cham. [https://doi.org/10.1007/978-3-031-14324-3\\_2](https://doi.org/10.1007/978-3-031-14324-3_2)

[26] Areias, P., Carapau, F., Lopes, J.C., Rabczuk, T. (2022). Consistent C Element-Free Galerkin Method for Finite Strain Analysis. In: Carapau, F., Vaidya, A. (eds) Recent Advances in Mechanics and Fluid-Structure Interaction with Applications. Advances in Mathematical Fluid Mechanics. Birkhäuser, Cham. [https://doi.org/10.1007/978-3-031-14324-3\\_6](https://doi.org/10.1007/978-3-031-14324-3_6)

### **Versão PT**

## **Proposta de Trabalho Científico para Tese de Doutoramento em Matemática**

### **Domínio Científico**

Matemática

### **Área**

Matemática da Mecânica dos Fluidos e suas Aplicações, Análise Matemática, Simulações Numéricas e Modelação Matemática com Experimentação

### **Tema Provisório para a Tese**

Problemas Unidimensionais de Interação Fluido-Estrutura usando a Teoria de Cosserat Theory

### **Palavras Chave**

Modelo Unidimensional, Problemas de Interação Fluido-Estrutura, Teoria de Cosserat, Fluidos Newtonianos, Fluidos não-Newtonianos, Equações Diferenciais Parciais, Equações Diferenciais Ordinárias, Experimentação Laboratorial

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### **Proposta de Trabalho Científico:**

Nas últimas décadas, o estudo dos fenômenos de interação entre fluido e estrutura tem atraído um grande número de pesquisadores de diversas áreas como engenharia mecânica, biologia, medicina, engenharia biomédica, física e matemática. No campo das aplicações em medicina, biologia e engenharia médica, os fenômenos de estrutura fluida têm recebido cada vez mais atenção na comunidade científica porque têm aplicação em uma ampla variedade de problemas associados ao fluxo de sangue no sistema vascular como, por exemplo, o problema clínico de uma situação de arteriosclerose ou veia aneurismática, entre outros. Hoje, o estudo dos fenômenos de estrutura e fluido é um tema de grande interesse, com grandes implicações sociais e econômicas. Os modelos a serem estudados estão relacionados ao problema específico associado à situação de arteriosclerose ou veia aneurismática, ou seja, envolvendo o fluxo de sangue, acoplado à estrutura e/ou permeabilidade da veia. Os modelos matemáticos mais recentes para este problema de interação fluido-estrutura encontrados na literatura são sofisticados, tendo sua gênese em uma ampla variedade de fatores, como escalas múltiplas e interações complexas entre fluido e o meio poroso. Esses modelos geralmente envolvem sistemas acoplados de equações diferenciais parciais (ou ordinárias) não lineares dependentes do tempo e equações algébricas. O seu tratamento analítico e numérico é complexo e requer o uso de argumentos sofisticados. Para ultrapassar este desvio, utilizamos uma teoria alternativa, a teoria de Cosserat associada à dinâmica dos fluidos, que nos permite aproximar o modelo 3D por modelos 1D simplificados. Com base nessa teoria de abordagem, reduzimos as equações tridimensionais não lineares que governam o movimento instável axissimétrico de um fluido incompressível não-newtoniano (ou fluido newtoniano) a um sistema unidimensional de equações diferenciais ordinárias (ou parciais) dependendo do tempo e de uma única variável espacial. A partir deste novo sistema, obtemos a equação instável para o gradiente de pressão médio e a tensão de cisalhamento na parede, ambas dependendo da vazão volumétrica, número de Womersley, viscosidade e parâmetros viscoelásticos em uma seção finita de um tubo reto com seção transversal circular variável.

O objetivo deste projeto é justamente estudar, em termos de análise matemática, otimização numérica e experiências laboratoriais, esses modelos alternativos resultantes da aplicação da teoria de Cosserat ao problema clínico da arteriosclerose ou situação de veia aneurismática.

#### **Algumas referências gerais sobre problemas de interação fluido-estrutura:**

- [1] K. J. Paik, Simulation of fluid-structure interaction for surface ships with linear/nonlinear deformations [Ph.D. thesis], University of Iowa, 2010.
- [2] Z. Mortazavinia, A. Zare, and A. Mehdizadeh, "Effects of renal artery stenosis on realistic model of abdominal aorta and renal arteries incorporating fluid-structure interaction and pulsatile non-Newtonian blood flow," *Applied Mathematics and Mechanics*, vol. 33, no. 2, pp. 165–176, 2012.
- [3] W. Zhang, C. Herrera, S. N. Atluri, and G. S. Kassab, "Effect of surrounding tissue on vessel fluid and solid mechanics," *Journal of Biomechanical Engineering*, vol. 126, no. 6, pp. 760–769, 2004.
- [4] V. Kanyanta, A. Ivankovic, and A. Karac, "Validation of a fluid-structure interaction numerical model for predicting flow transients in arteries," *Journal of Biomechanics*, vol. 42, no. 11, pp. 1705–1712, 2009.
- [5] P. K. Siogkas, A. I. Sakellarios, T. P. Exarchos et al., "Blood flow in arterial segments: rigid vs. deformable walls simulations," *Journal of the Serbian Society for Computational Mechanics*, vol. 5, no. 1, pp. 69–77, 2011.
- [6] M. H. Friedman, R. Krams, and K. B. Chandran, "Flow interactions with cells and tissues: cardiovascular flows and fluid-structure interactions: sixth international bio-fluid mechanics symposium and workshop, March 28–30, 2008, Pasadena, California," *Annals of Biomedical Engineering*, vol. 38, no. 3, pp. 1178–1187, 2010.
- [7] K. W. Lee and X. Y. Xu, "Modelling of flow and wall behaviour in a mildly stenosed tube," *Medical Engineering and Physics*, vol. 24, no. 9, pp. 575–586, 2002.
- [8] S. Badia, A. Quaini, and A. Quarteroni, "Splitting methods based on algebraic factorization for fluid-structure interaction," *SIAM Journal on Scientific Computing*, vol. 30, no. 4, pp. 1778–1805, 2008.
- [9] G. Guidoboni, R. Glowinski, N. Cavallini, S. Čanić, and S. Lapin, "A kinematically coupled time-splitting scheme for fluid-structure interaction in blood flow," *Applied Mathematics Letters*, vol. 22, no. 5, pp. 684–688, 2009.

[10] F. Nobile and C. Vergara, "An effective fluid-structure interaction formulation for vascular dynamics by generalized Robin conditions," SIAM Journal on Scientific Computing, vol. 30, no. 2, pp. 731–763, 2008.

### **Algumas Referências Preparatórias Obras do Orientador, Coorientador e do**

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[1] Carapau, F., and Sequeira, A., 1D Models for Blood Flow in Small Vessels Using the Cosserat Theory, WSEAS Transactions on Mathematics, Issue 1, Volume 5, pp. 54-62, 2006.

[2] Carapau, F., and Sequeira, A., Axisymmetric motion of a second order viscous fluid in a circular straight tube under pressure gradients varying exponentially with time, WIT Transactions on Engineering Sciences, Vol.52, pp. 409-419, 2006.

[3] Carapau, F., and Sequeira, A., Unsteady flow of a generalized Oldroyd-B fluid using a director theory approach, WSEAS Transactions on Fluid Mechanics, Issue 2, Volume 1, pp. 167-174, 2006.

[4] Janela, J., Sequeira, A., and Carapau, F., Numerical Simulation of the Motion of Rigid Particles in Generalized Newtonian Fluids using a Hyper-Viscosity Method, WSEAS Transactions on Mathematics, Issue 4, Volume 5, pp. 366-373, 2006.

[5] Carapau, F., Sequeira, A., and Janela, J., 1D simulations of second-grade fluids with shear-dependent viscosity, WSEAS Transactions on Mathematics, Issue 1, Volume 6, pp. 151-158, 2007.

[6] Carapau, F., and Sequeira, A., 1D dynamics of a second-grade viscous fluid in a constricted tube, Banach Center Publications, Inst. Math, Polish Acad. Sc., Volume 81, pp. 95-103, 2008.

[7] Carapau, F., Axisymmetric motion of a generalized Rivlin-Ericksen fluids with shear-dependent normal stress coefficients, Inter. Journal of Mathematical Models and Methods in Applied Sciences, Issue 2, Volume 2, pp. 168-175, 2008.

[8] Carapau, F., Analysis of perturbed flows of a second-order fluid using a 1D hierarchical model, Inter. Journal of Mathematics and Computers in Simulation, Issue 3, Volume 2, pp. 256-263, 2008.

[9] Carapau, F., Axisymmetric Swirling Motion of Viscoelastic Fluid Flow Inside a Slender Surface of Revolution, IAENG Engineering Letters, Issue 4, Volume 17, pp. 238-245, 2009.

[10] Carapau, F., 1D Viscoelastic Flow in a Circular Straight Tube with Variable Radius, *Int. J. Appl. Math. Stat.*, No. D10, Volume 19, pp. 20-39, 2010.

[11] Carapau, F., One-dimensional viscoelastic fluid model where viscosity and normal stress coefficients depend on the shear rate, *Nonlinear Analysis: Real World Applications*, Volume 11, pp. 4342-4354, 2010.

[12] Carapau, F., and Janela, J., A one-dimensional model for unsteady axisymmetric swirling motion of a viscous fluid in a variable radius straight circular tube. *International Journal of Engineering Science*, Volume 72, pp. 107-116, 2013.

[14] Carapau, F., and Correia, P., Numerical simulations of a third-grade fluid flow on a tube through a contraction, *European Journal of Mechanics B/Fluids*, V. 65, pp. 45-53, 2017.

[15] Fernando Carapau, Paulo Correia, Luís M. Grilo and Ricardo Conceição, Axisymmetric Motion of a Proposed Generalized Non-Newtonian Fluid Model with Shear-dependent Viscoelastic Effects, *IAENG International Journal of Applied Mathematics*, vol. 47, no. 4, pp. 361-370, 2017.

[16] Carapau, F., Janela, J., Correia, P., Vila, S., Numerical Solvability of a Cosserat Model for the Swirling Motion of a Third-grade Fluid in a Constant Radius Straight Circular Tube, *Int. J. Appl. Math. Stat.*, vol. 57, no. 2, pp. 1-15, 2018.

[17] Carapau, F., Conceição, R., Three-Dimensional Velocity Field for Blood Flow Using the Power-Law Viscosity Function, *WSEAS Transactions on Heat and Mass Transfer*, V.13, pp. 35-48, 2018.

[18] P. Areias, T. Rabczuk, F. Carapau and J. Carrilho Lopes, A continuous-stress tetrahedron for finite strain problems, *Finite Elements in Analysis and Design*, vol. 165, pp. 52-64, 2019.

[19] P. Areias, C. Tiago, J. Carrilho Lopes, F. Carapau, P. Correia, A finite strain Raviart-Thomas tetrahedron, *European Journal of Mechanics / A Solids*, V. 80 (2020) 103911 (doi.org/10.1016/j.euromechsol.2019.103911)

[20] F. Carapau, P. Correia, T. Rabczuk and P. Areias, One-dimensional model for the unsteady flow of a generalized third-grade viscoelastic fluid, *Journal Neural Computing & Applications*, (DOI: 10.1007/s00521-020-04733-w)

[21] Corrigendum to “On the modified least action principle with damping” [*Eur. J. Mech. B Fluids* 89 (2021) 301-311](S099775462100090X)(10.1016/j.euromechflu.2021.06.005), Fitzgerald, K., Massoudi, M. & Vaidya, A., 1 Jan 2022, In: *European Journal of Mechanics, B/Fluids*. 91

[22] Network Thermodynamics-Based Scalable Compartmental Model for Multi-Strain Epidemics, Pateras, J., Vaidya, A. & Ghosh, P., Oct 2022, In: Mathematics. 10, 19, 3513

[23] On the Thermodynamics of Self-Organization in Dissipative Systems: Reflections on the Unification of Physics and Biology, Chung, B. J., De Bari, B., Dixon, J., Kondepudi, D., Pateras, J. & Vaidya, A., Apr 2022, In: Fluids. 7, 4, 141

[24] Ramos, C., Carapau, F., Correia, P. (2022). Cellular Automata Describing Non-equilibrium Fluids with Non-mixing Substances. In: Carapau, F., Vaidya, A. (eds) Recent Advances in Mechanics and Fluid-Structure Interaction with Applications. Advances in Mathematical Fluid Mechanics. Birkhäuser, Cham. [https://doi.org/10.1007/978-3-031-14324-3\\_10](https://doi.org/10.1007/978-3-031-14324-3_10)

[25] Carapau, F., Correia, P., Areias, P. (2022). Three-Dimensional Velocity Field Using the Cross-Model Viscosity Function. In: Carapau, F., Vaidya, A. (eds) Recent Advances in Mechanics and Fluid-Structure Interaction with Applications. Advances in Mathematical Fluid Mechanics. Birkhäuser, Cham. [https://doi.org/10.1007/978-3-031-14324-3\\_2](https://doi.org/10.1007/978-3-031-14324-3_2)

[26] Areias, P., Carapau, F., Lopes, J.C., Rabczuk, T. (2022). Consistent C Element-Free Galerkin Method for Finite Strain Analysis. In: Carapau, F., Vaidya, A. (eds) Recent Advances in Mechanics and Fluid-Structure Interaction with Applications. Advances in Mathematical Fluid Mechanics. Birkhäuser, Cham. [https://doi.org/10.1007/978-3-031-14324-3\\_6](https://doi.org/10.1007/978-3-031-14324-3_6)