Discrete dynamical systems with delay: iterated maps on 2D domains.

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Description:

In this work, we propose the study of discrete dynamical systems from the plane to the plane through one-dimensional delay systems.

Delay dynamical systems are characterized by the future behavior of a variable depending not only on the current state but also on past states, with a certain delay. The study and analysis of these systems can be conducted through various techniques such as stability analysis, bifurcation theory, symbolic dynamics, kneading theory, Markov partitions, and numerical simulation, aiming to understand how the delay affects the global behavior of the system.

The proposed work will focus on the analysis of iterations of plane-to-plane maps of the form

$$F_{a,b}(x,y) = \Big(f_{a,b}(x,y), g_{a,b}(x,y)\Big),$$

Where a and b real constants, and $f_{a,b}$ and $g_{a,b}$ can be linear or nonlinear. We will particularly explore the applications of Lozi and Hénon, defined by $L_{a,b}(x,y) = (1 - a|x| + y, bx)$ e $H_{a,b}(x,y) = (1 - ax^2 + y, bx)$ respectively.

Student profile: Prerequisite for ODE I, ODE II, Dynamical Systems, Numerical Analysis, and Python Programming.

References:

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[4] Y. Ishii, Towards a kneading theory for Lozi mappings I. A solution of the pruning front conjecture and the first tangency problem, Nonlinearity 10 (1997), no. 3, 731–747.

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